Imperial College London

DE2 Electronics 2

Lecture 17

Revision Lecture

Peter Cheung

Dyson School of Design Engineering

URL: www.ee.ic.ac.uk/pcheung/teaching/DE2_EE/
E-mail: p.cheung@imperial.ac.uk

2018 Exam Paper – Q1 Basic Signals

1. An electrical signal is represented mathematically by the equation:

$$x(t) = 2.35\sin(3142t + 30°) + 0.65 \text{ volts}$$

~1min/mark

(i) What is the average value of $x(t)$? 0.65v

[2]

(ii) What is the frequency in Hz and phase angle in radians of the signal?

500Hz, $\frac{\pi}{6}\ rad/\sec at\ t = 0$

[3]

(iii) What are its maximum and minimum values?

[4]

3v and -1.7v

2017 Exam Question 1 – Basic signals

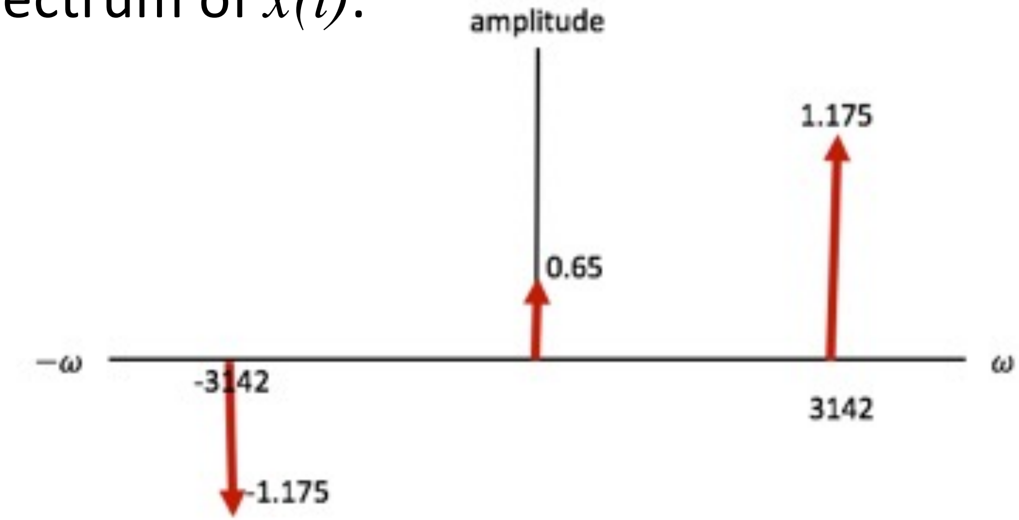
$$x(t) = 2.35\sin(3142t + 30°) + 0.65 \text{ volts}$$

(iv) Rewrite x(t) in exponential instead of sinusoidal form.

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$x(t) = 0.65 + \frac{1.175}{j}\left(e^{j(3142t+\frac{\pi}{6})} - e^{-j(3142t+\frac{\pi}{6})}\right)$$

(v) Sketch the amplitude spectrum of *x(t)*.



2017 Exam Question 2 – Sampling & ADC

2. The signal in Q1 is to be sampled by a microprocessor system with an analogue to digital converter (ADC).

   (i) What is the minimum sampling frequency that you must use in order not to corrupt the converted signal? In practice, what sampling frequency would you choose to use and why?

Fs(min) = 1kHz,

2.5 x Fs(min) or higher, i.e. 2.5kHz.

   (ii) It is known that the converted digital signal requires an accuracy of 0.1%. How many bits of resolution is required of the ADC to achieve this accuracy?

requires 10 bits ADC – 1 part in 1024.

2018 Exam Question 1 (i) – Sampling & ADC

1. You are designing an electronic system to measure breathing of a patient with a breathing frequency of up to 40 breaths/minute. However, the diagnostic technique measures turbulence in the flow that would be up to 100 times faster than the breathing frequency. Furthermore, the medical team told you that they need your system to have a measurement accuracy of 0.05%. You are required to pick an analogue to digital converter (ADC) to turn your transducer measurements, which ranges from 0 to 3.3V, to digital samples.

    (i) What would you choose as the sampling frequency for the ADC? Why?

Breathing rate is 40/60 Hz, and we want to capture signals at least 100 times faster. Maximum frequency is about 66.7Hz. Sampling theorem dictates that the sampling rate must be at least 2x of maximum frequency. So, must use sampling rate of 133Hz at least. However, for easy reconstruction, we usually use a practical sampling rate a few times higher than this (say 2.5x) at just over 300Hz. Accept well justified answers.

2018 Exam Question 1 (ii) – Sampling & ADC

1. You are designing an electronic system to measure breathing of a patient with a breathing frequency of up to 40 breaths/minute. However, the diagnostic technique measures turbulence in the flow that would be up to 100 times faster than the breathing frequency. Furthermore, the medical team told you that they need your system to have a measurement accuracy of 0.05%. You are required to pick an analogue to digital converter (ADC) to turn your transducer measurements, which ranges from 0 to 3.3V, to digital samples.

   (ii) How many bits must the ADC have as converted values? Why? What is the resolution of the ADC in volts?

0.05% = 1 in 2000. Therefore, minimum number of bits in ADC is 11-bits (2^{11}). The resolution of the ADC in volts would be 3.3V/2048 = 1.61mV.

2018 Exam Question 1 (iii) – Anti-aliaising

1. You are designing an electronic system to measure breathing of a patient with a breathing frequency of up to 40 breaths/minute. However, the diagnostic technique measures turbulence in the flow that would be up to 100 times faster than the breathing frequency. Furthermore, the medical team told you that they need your system to have a measurement accuracy of 0.05%. You are required to pick an analogue to digital converter (ADC) to turn your transducer measurements, which ranges from 0 to 3.3V, to digital samples.

(iii) You were also told that the transducer could pick up interference of unknown frequencies from its surrounding. State with justifications and assumptions how you may avoid your captured signal being corrupted by such interference.

To prevent corrupting our base-band signal due to aliasing effect or frequency folding due to sampling, we need to first filter the signal with an anti-aliasing filter. If we choose a sampling frequency of 1kHz, then we need to use a lowpass filter on the breathing signal before sampling that cut out all signals (at least 40dB attenuation) for signals above 0.5kHz.

2019 Exam Paper – Q1 Another Signals Question

1. A signal $x(t)$ can be modelled mathematically as:

$$x(t) = \left[2.5 + 1.5\cos(31.42t + \tfrac{\pi}{4})\right] + 0.5\,\delta(t-1)$$

(i) Sketch the waveforms $1.5\cos(31.42t + \frac{\pi}{4})$ and $0.5\,\delta(t-1)$ for $0 \le t \le 0.2$. Hence sketch the signal $x(t)$ for $0 \le t \le 0.2$.

[8]

(ii) Rewrite $x(t)$ in exponential form where appropriate. (There is no need to simplify the equation.)

[4]

(iii) Sketch the amplitude spectrum $|X(j\omega)|$ of the signal x(t).

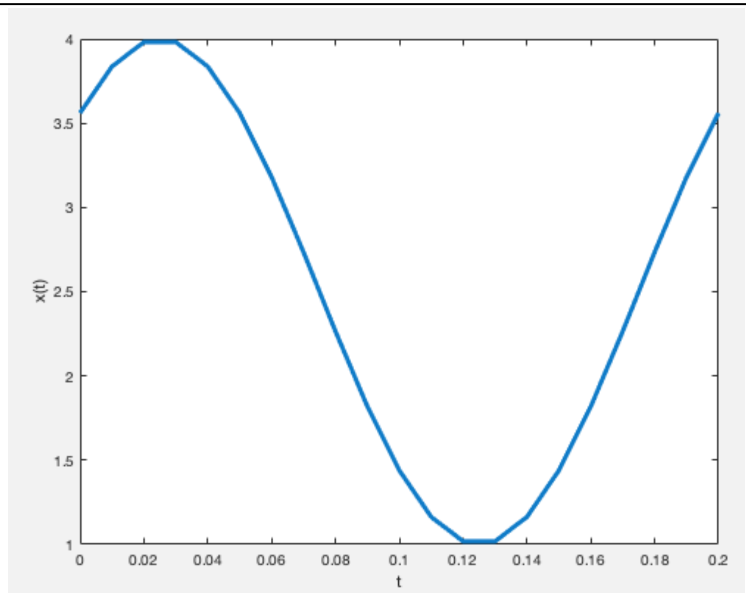
[4]

2019 Exam Paper – Q1 (i) Solution

$$x(t) = \left[2.5 + 1.5\cos(31.42t + \tfrac{\pi}{4})\right] + 0.5\,\delta(t-1)$$

(i) Sketch the waveforms $1.5\cos(31.42t + \frac{\pi}{4})$ and $0.5\,\delta(t-1)$ for $0 \le t \le 0.2$. Hence sketch the signal $x(t)$ for $0 \le t \le 0.2$.

The signal has three components: a cosine signal at 5Hz with a 45 degrees delay, a 3.5v dc offset, and an impulse of 0.5v at t=1. Students are expected to produce a rough sketch of x(t) of the cosine signal. However, the impulse is outside range of t, so can be omitted.



2019 Exam Paper – Q1 (ii) Solution

$$x(t) = \left[2.5 + 1.5\cos(31.42t + \tfrac{\pi}{4})\right] + 0.5\,\delta(t-1)$$

(ii) Rewrite $x(t)$ in exponential form where appropriate. (There is no need to simplify the equation.)

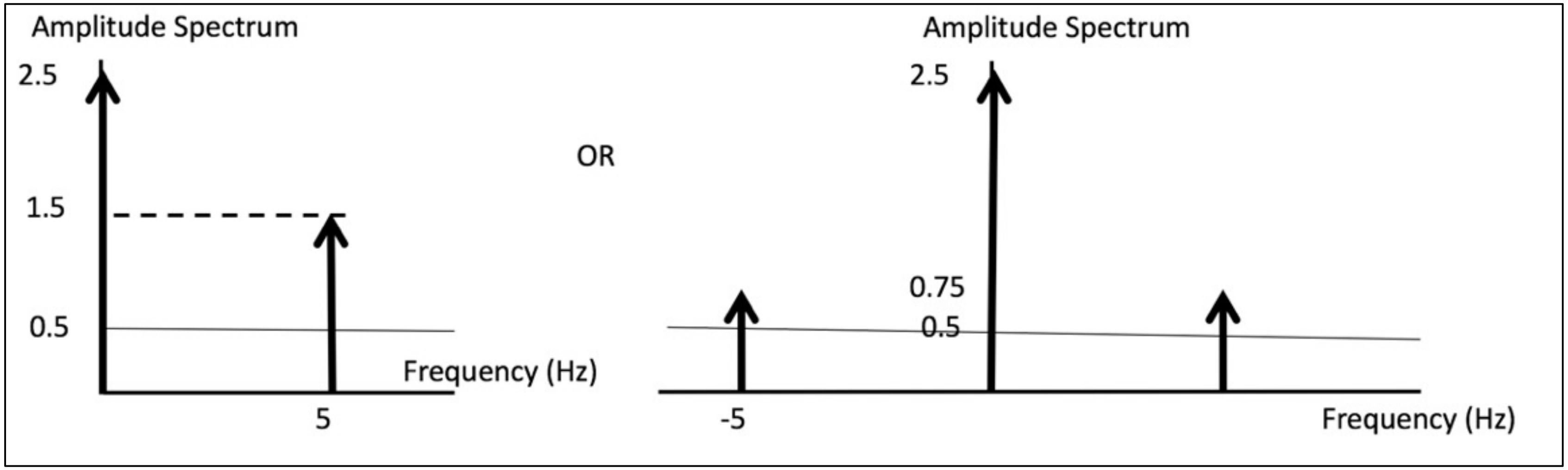
Straight forward application of Euler's formula:

$$x(t) = 0.75\left(e^{+j(31.42+\pi/4)} + e^{-j(31.42j+\pi/4)}\right) + 2.5 + 0.5\delta(t-1)$$

2019 Exam Paper – Q1 (iii) Solution

$$x(t) = \left[2.5 + 1.5\cos(31.42t + \tfrac{\pi}{4})\right] + 0.5\,\delta(t-1)$$

(iii) Sketch the amplitude spectrum $|X(j\omega)|$ of the signal x(t).



Sample paper Q3 – Motor sensor, Polling vs interrupt

3. Figure Q3 shows a motor with a hall effect sensor detecting the rotational speed of a motor. The circular magnet attached to the motor's axle has 12 pairs of magnetic poles. The output of the hall effect sensor produces a series of pulses, one pulse for every N-S pair of the circular magnet, which is counted by the microprocessor system.

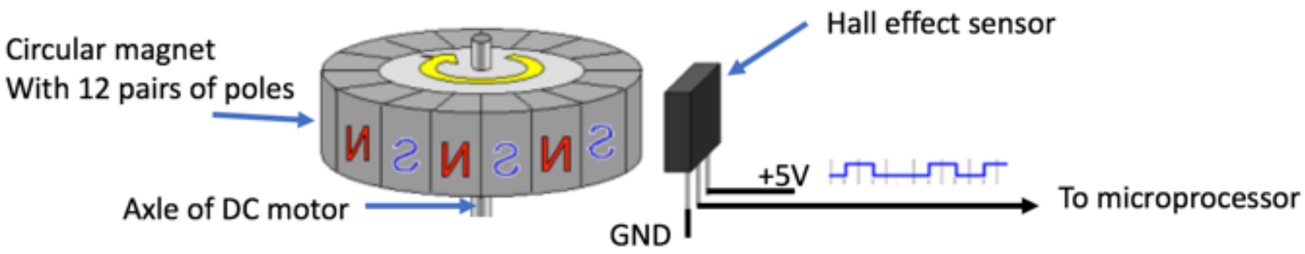
   (i) If C is the number of pulses counted over a period of 100 msec, write down the equation relating the speed of the motor S in revolution/minute (rpm) to the pulse count C.

   $$S = \left(\frac{C}{12} \times 10\right) \times 60 \text{ rpm}$$

   [6]

   (ii) The pulses could be counted by the microprocessor using the method of poling or interrupt. Explain in no more than 100 words the advantages and disadvantages of these two methods.

   [6]



2018 Exam Q4 – Differential equation & Laplace Transform

4. The following differential equation describes the relationship between the output y(t) and the input x(t) of a linear system:

$$7\frac{d^2y}{dt^2} + \frac{dy}{dt}10 = \frac{d^2x}{dt^2} + 2\frac{dx}{dt} - 4y(t)$$

(i) What is the order of this system? [2]

This is a second order system.

(ii) Given that Y(s) and X(s) are the Laplace Transforms of y(t) and x(t) respectively, write down the transfer function H(s) = Y(s)/X(s) for the system. [4]

$$H(s) = \frac{s^2 + 2s}{7s^2 + 10s + 4}$$

2018 Exam Q5 (i) & (ii) – Transfer function

5. A system H consists of two circuits A and B connected in series with each other as shown in Figure Q5. The transfer function for circuit A is P(s) and for circuit B is Q(s), and they are known to be:

$$P(s) = \frac{1}{0.5s + 1} \qquad Q(s) = \frac{100}{s^2 + 2ks + 100}$$

where k is a constant.

(i) Derive the s-domain equation for the transfer function H(s) of the entire system?

$$H(s) = P(s)Q(s) = \frac{100}{0.5s^3 + (k+1)s^2 + (2k+50)s + 100}$$

(ii) What is the natural or resonant frequency of the system?

$$K \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

The resonant frequency is determined by the Q(s)

$\sqrt{100}$ = 10 rad/sec.

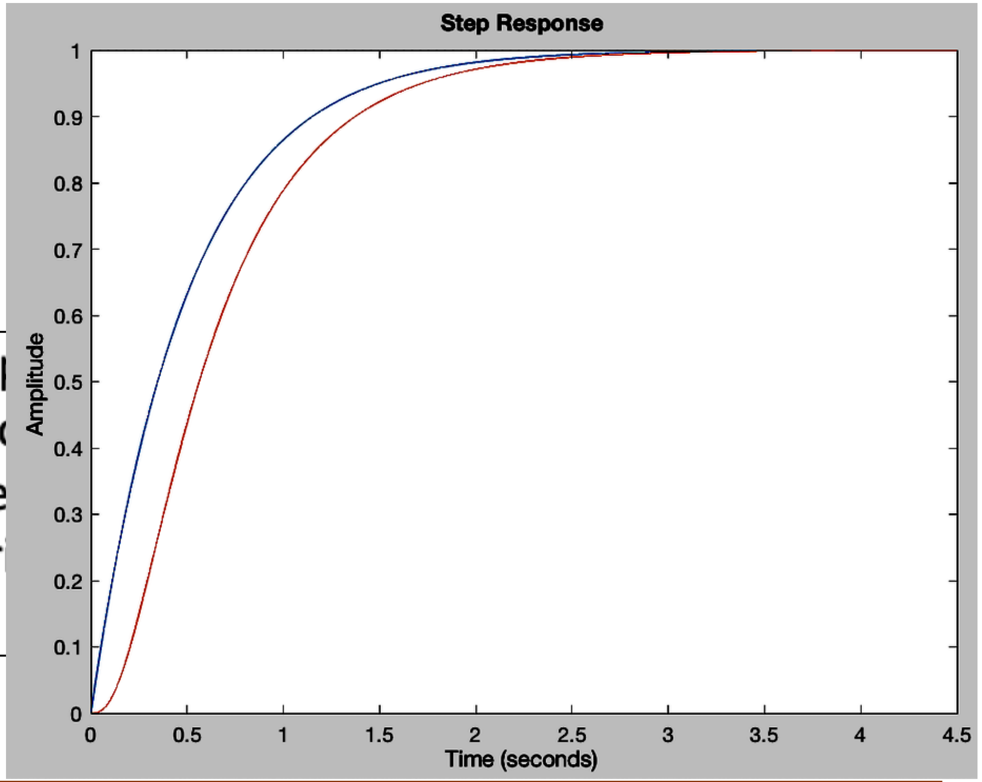
2018 Exam Q5 (iii) – Step Response & critical damping

$$P(s) = \frac{1}{0.5s + 1} \qquad Q(s) = \frac{100}{s^2 + 2ks + 100}$$

$$K\frac{{\omega_0}^2}{s^2 + 2\zeta\omega_0 s + {\omega_0}^2}$$

(iii) It is known that when k = 10, the system is critically damped. Sketch the step response of the system. Explain your answer.

When the system is critically damped, the step dominated by the first-order system P(s), whic constant of 0.5sec (shown in blue). Therefore response is approximated by an exponential r roughly 63% at 0.5s. (Shown in blue).



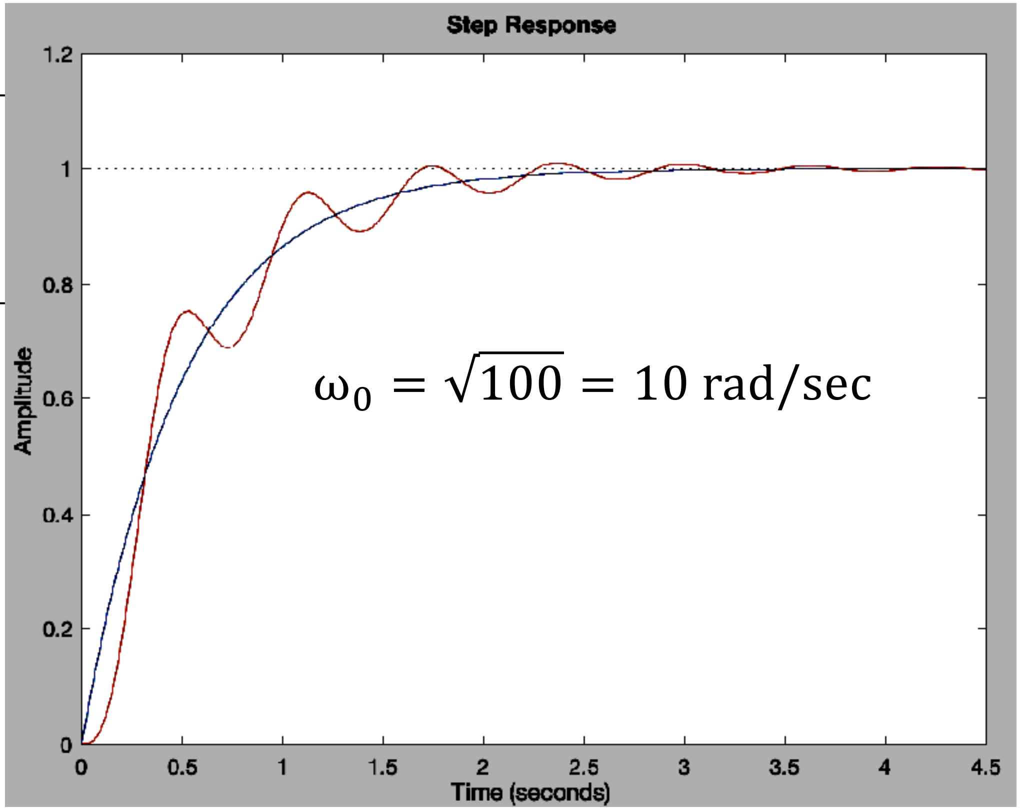
2018 Exam Q5 (iv) – Step Response & underdamping

$$P(s) = \frac{1}{0.5s + 1} \qquad Q(s) = \frac{100}{s^2 + 2ks + 100}$$

$$K\frac{\omega_0{}^2}{s^2 + 2\zeta\omega_0 s + \omega_0{}^2}$$

(iv) If k is 1, sketch the step response of the system. Explain your answer.

When k = 1, the system is underdamped, and therefore you will see oscillation at the natural frequency, which is around 1.6Hz.

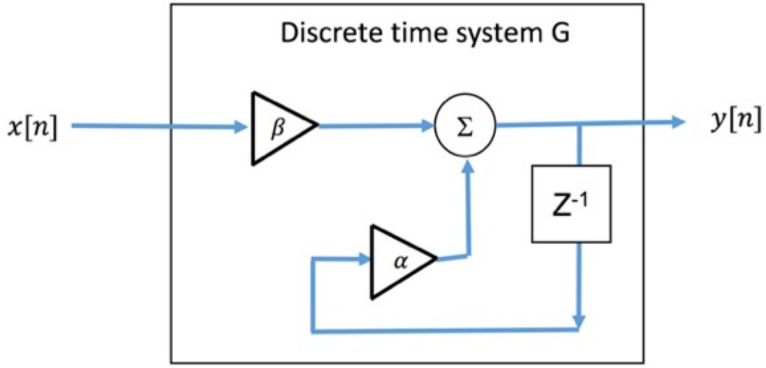


$$\omega_0 = \sqrt{100} = 10 \text{ rad/sec}$$

2018 Exam Question 6 (i) & (ii) – Discrete signals and z-transform

6. The block diagram of a discrete-time shift-invariant system G is shown in Figure Q6. The input to the system is $x[n]$ and the output is $y[n]$, where $n = 0,1,2,3, \ldots$ etc. The system is assumed to be casual, i.e. $x[n] = y[n] = 0,$ for $n < 0$.

(i) Derive the difference equation for the system.



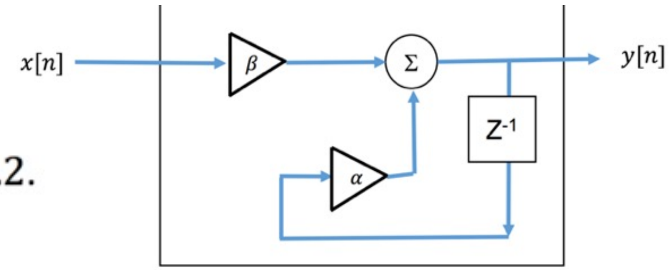
$$y[n] = \alpha y[n-1] + \beta x[n]$$

(ii) Derive the output sequence $y[0], y[1], \ldots \ldots \ldots, y[9]$ given that $\alpha = 0.8, \beta = 0.2$, $x[0] = 0$ and $x[n] = 10$ for $n \geq 1$.

n	0	1	2	3	4	5	6	7	8	9
x[n]	0	10	10	10	10	10	10	10	10	10
y[n]	0	2	3.6	4.88	5.9	6.72	7.38	7.9	8.32	8.66

2018 Exam Question 6 (iii) –
z-transform and z-domain transfer function

6. The block diagram of a discrete-time shift-invariant system G is shown in Figure Q6. The input to the system is $x[n]$ and the output is $y[n]$, where $n = 0,1,2,3, \ldots$ etc. The system is assumed to be casual, i.e. $x[n] = y[n] = 0$, for $n < 0$.



(iii) Find the transfer function $G[z]$ of the system for $\alpha = 0.8, \beta = 0.2$.

$$y[n] = \alpha y[n-1] + \beta x[n]$$

Take z-transform on both sides of the difference equation:

$$Y(z) = 0.8z^{-1}Y(z) + 0.2X(z)$$

$$\Longrightarrow (1 - 0.8z^{-1})Y(z) = 0.2X(z)$$

$$G(z) = \frac{Y(z)}{X(z)} = \frac{0.2}{1 - 0.8z^{-1}} = \frac{0.2z}{z - 0.8}$$

2019 Exam Question 6 (i) – z-transform and z-domain transfer function

6. A 5-tap moving average filter has discrete output signal y[n] and input signal x[n], and the system is causal. The filter has a difference equation given by:

$$y[n] = \frac{1}{5}(x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4])$$

(i) If Y(z) and X(z) are the z-transform of the discrete signals y[n] and x[n] respectively, derive the transfer function Y(z)/X(z).

$$H(z) = \frac{Y(z)}{X(z)} = 0.2(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$$

2019 Exam Question 6 (ii) – z-transform and z-domain transfer function

$$H(z) = \frac{Y(z)}{X(z)} = 0.2(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$$

(ii) Sketch, not necessary to scale, the frequency response you expect of such a filter.